Friday 13 July 2012 Exam I 60 minutes

Name:

Instructions:

This is the first real test of the course. The material is entirely from the first four days of class, covering Appendix B, sections 1.1-1.6, and sections 2.1-2.5. Be calm and think through each question - several have 'trick' parts. Do not be afraid to skip and come back to different questions or to ask for help/clarifications, and note that it is not necessarily true that questions get harder as the test progresses.

(1) Do your (own) best work, and draw a box around your final answers.
(2) Extra paper is available if you need it.
(3) Show all the steps of your work clearly and label everything so that anyone who knows the material would be able to understand the answer.
(4) This exam is a calculator-exam – you may use your calculator on every question.
(5) Ask any questions or raise any concerns whenever you have any doubt.

Question	Points	Your Score
Q1	15	
Q2	10	
Q3	10	
Q4	15	
Q5	10	
Q6	20	
Q7	20	
TOTAL	100	

[Q1]. . . [15 points]

[*1*] Consider the following three points: $A = (8, 0), \quad B = (4, 2), \quad C = (-2, -10)$.
On a sheet of graph paper, draw and label a set of axes, and draw and label these three points.
Show that the triangle ABC is a right triangle.

[*2*] A new triangle $a'b'c'$ is formed from ABC as follows: first, let c be the midpoint between A and B, a be the midpoint between B and C, and b is the midpoint between A and C. (draw and label these on the graph from above). Then the triangle abc is shifted down by 3 units and left by 1 unit to give the triangle $a'b'c'$.

In class, we showed that this triangle will also be a right triangle. Verify the Pythagorean Theorem on the triangle $a'b'c'$.

[*3*] Write the equation for a circle around the origin of radius 4.

[*4*] What is the equation for a circle around the point $(1, 2)$ of radius 2?

[Q2]. . . [10 points] Solve the following equations for x:

[*1*] $2x^2 = 19x + 33$

[*2*] $(x + 2)^2 = x^2 - 6x + 1$

[*3*] $4x^3 + 12x^2 - 8x - 24 = 0$

[*4*] $\sqrt{x - 10} - 4 = 0$

[*5*] $\sqrt{x} - \sqrt{x - 5} = 1$

[Q3]. . . [10 points] Solve the following inequalities for x. Be sure to clearly indicate which x are the solutions, and use the bracket-style notation we use in class.

[1] $x^2 - 4x - 5 \geq 0$

[2] $|5x| > 10$

[3] $x^4(x - 3) < 0$

[4] $4x^2 + 12x + 5 > -4$

[Q4]. . . [15 points]

[*1*] A line ℓ passes through the points $(7, -1)$ and $(10, 4)$. What is the equation of that line? On a sheet of graph paper, draw and label this line.

[*2*] What is the equation for the line ℓ' that is perpendicular to ℓ and goes through the point $(1, 1)$? Draw this on the graph from above.

[*3*] What is the domain and range of $f(x) = 2\sqrt{x-3} + 2$?

[*4*] What is the domain and range of $g(x) = \sqrt{49 - x^2} - 1$?

[*5*] On graph paper, carefully sketch (it doesn't have to be exact, but try to get the idea) the function $\lfloor (x-1)^2 + 1 \rfloor$ for x in $[-1, 3]$.

[Q5]. . . [10 points]

[*1*] Let $f(x) = x^2$ and $g(x) = \sqrt{x-3}$. What are the domain and range of f and g?

[*2*] Find the following, and specify their domain (but not their range):

(1) $(f-g)(x)$

(2) $\left(\frac{f}{g}\right)(x)$

(3) $f \circ g(x)$

(4) $g \circ f(x)$

This is a problem that guides you through a calculus concept. Still consider $f(x) = x^2$, find $f(x+h)$. Then find $\frac{f(x+h)-f(x)}{h}$ and simplify it as much as possible.

Without considering domain problems, consider the function $f'(h) = \frac{f(x+h)-f(x)}{h}$, a function of h and not x. What is $f'(0)$ (in terms of x)?

[Q6]. . . [20 points] Function Classification

[*1*] Classify the function $h(x) = \frac{1}{4}x^4 - 2x^2$

[*2*] Classify the function $g(x) = |x + 2| - |x - 2|$

[Q7]. . . [20 points]

[*1*] Completely factor (both in the real and complex sense) the function $f(x) = x^3 + 10x^2 + 33x + 34$

[*2*] Find all roots (real and complex) of the function $g(x) = x^4 + 6x^3 + 10x^2 + 6x + 9$