Friday 20 July 2012 Exam II 60 minutes

Name:

Instructions:

This is the second test of the course. This test is focused on the material covered since the last exam, meaning the end of chapter 2, all of chapter 3, and sections 4.1-4.6. Be calm and think through each question - some may have 'trick' parts. Do not be afraid to skip and come back to different questions or to ask for help/clarifications, and note that it is not necessarily true that questions get harder as the test progresses.

(1) Do your (own) best work, and draw a box around your final answers.
(2) Extra paper is available if you need it.
(3) Show all the steps of your work clearly and label everything so that anyone who knows the material would be able to understand the answer.
(4) This exam is a calculator-exam – you may use your calculator on every question.
(5) Ask any questions or raise any concerns whenever you have any doubt.

Question	Points	Your Score
Q1	20	
Q2	15	
Q3	15	
Q4	25	
Q5	25	
TOTAL	100	

[Q1]. . . [20 points]

This question focuses on the rational function

$$f(x) = \frac{3x^3 - 11x^2 + 8x + 4}{4x^2 - 10x + 4}$$

[*1*] Start by considering the numerator. For what x is $3x^3 - 11x^2 + 8x + 4 = 0$? *HINT:* $x = -\frac{1}{3}$ *is a root.*

[*2*] Now consider the denominator. Solve $4x^2 - 10x + 4 = 0$.

[*3*] On a sheet of graph paper, draw $f(x)$. Be sure to clearly label asymptotes, zeroes, axes, scale, and anything else that would facilitate the reading of the graph.

[Q2]. . . [15 points] In this question, we will re-derive the logarithm change-of-base formula and use it.

[1] **Change of Base** Suppose that we know $\log_b(x)$ and that we want to know $\log_a(x)$. To change base, let's do the following:

(1) Start with the value we want to know. Suppose $\log_a(x) = y$. What is the equivalent statement in exponents? Write it down.
(2) Using the last part, let's now take $\log_b$ of both sides. Solve for y.
(3) Remembering that $\log_a(x) = y$, conclude that $\log_a(x) = \dfrac{\log_b(x)}{\log_b(a)}$

[2] Evaluate the following logarithms to three decimal places using the change-of-base formula:

(1) $\log_8 22$
(2) $\log_5 22$

[3] Solve the following equation: $2\log_{10}(x) - \log_{10}(15 - 5x) = 2$

[Q3]. . . [15 points] In this question, we use an exponential model and review properties of the logarithmic and exponential functions.

The evil organization Men Opposed to Math (MOM) gathered 100 porcupines and put them into a game preserve, waiting for their numbers to multiply so that they can throw them at good, innocent mathematicians. A kidnapped mathematician estimated that the growth of the prickle (*this is the real word for a group of porcupines*) follows the equation:

$$p(t) = \frac{1000}{1 + 9e^{-0.1656t}}$$

[*1*] What is the population of the prickle after 5 months.

[*2*] After how many months will the population reach 500 porcupines?

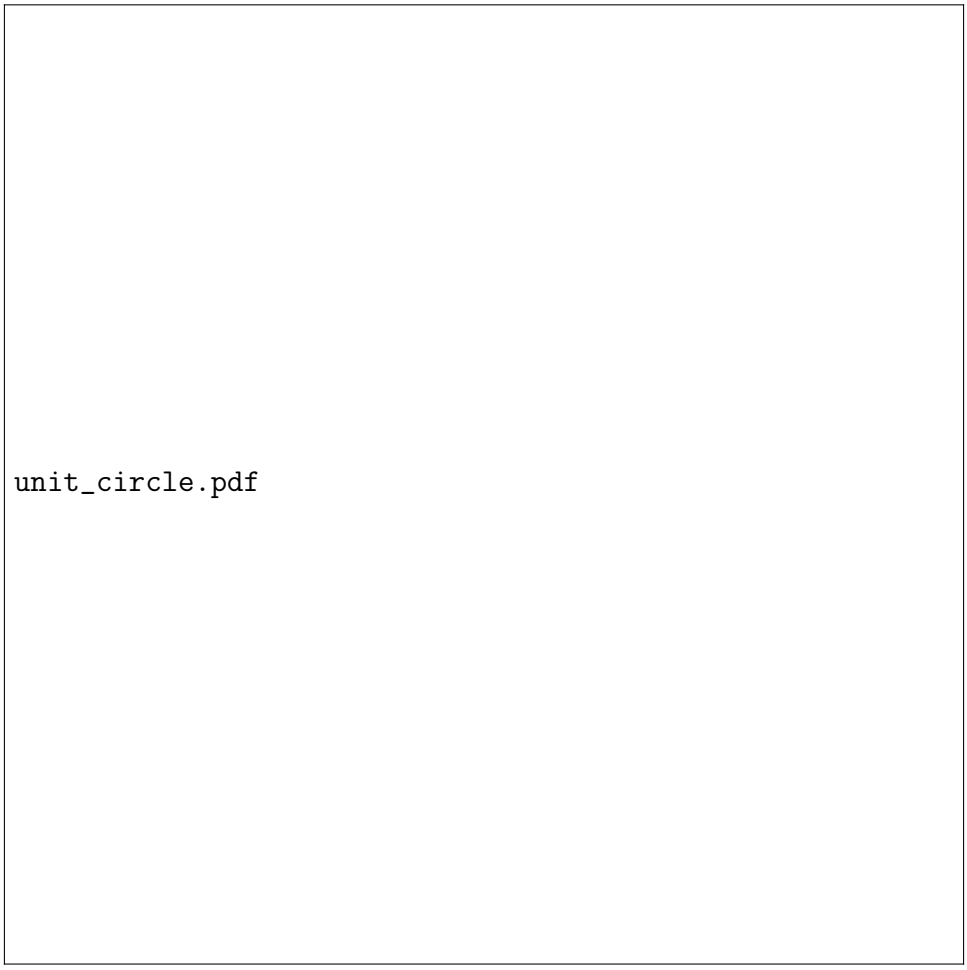
[*3*] Little known to MOM, all mathematicians have a secret love of all mammals with PI in their name, like porcuPInes, and have been planning on contributing to the newly build Museum of Mathematics (MoMath) in New York by dedicating a life exhibit. (Later, it will have other animals, like PIgs and caterPIllars). Being clever investors, they put in 5000 dollars in 1990 into an account with 8% interest, compounded quarterly (4 times a year). How much money will be ready for the exhibit by the year 2013?

[*3*] Will MOM succeed in disrupting the mathematicians?

yes no

[Q4]. . . [25 points] The Unit Circle and its applications

[*1*] Below, you will find a unit circle. Relabel the angles in radians, and write the sine and cosine values at each angle.



[*2*] What is $\sec \frac{2\pi}{3}$?

[*3*] Convert $\frac{\pi}{5}$ radians to degrees.

[*4*] Convert 12° to radians.

[Q5]. . . [25 points] This page is on basic trigonometry:

[1] A mathematician is trying to determine the width of a river. He stands across from a tree on the opposite bank of the river and walks 125 feet along the river, upstream. He pulls out a compass and measures the angle between his start position and the tree on the other side of the river: it's $62°$. How wide is the river?

[2] In a certain right triangle, it's known that $\cos(\theta) = 1/3$ and the hypotenuse is of length 21. Draw and label all sides and angles of this triangle.

[3] On a sheet of graph paper, clearly draw 3 periods of the function $f(x) = 1 + \tan(2x)$.

[4] On a sheet of graph paper, clearly draw the following two functions on the same set of axes:

- $f(x) = 2\csc(x)$
- $g(x) = 2\cos(2x - \pi)$

Use this graph to find the solutions to the equation $f(x) = g(x)$.